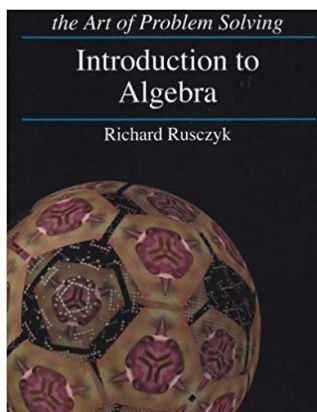


Algebra I (AoPS) – Class Description



Textbook and Workbook

Introduction to Algebra, 2nd Edition, by Richard Rusczyk, the Art of Problem Solving series.

Prerequisite

For seven graders or above, or advanced six graders with basic foundations in Algebra. Students should have passed the course of Pre-Algebra, and be proficient in fundamentals of Arithmetic, such as integers, decimals and fractions, primes and composites, addition and subtraction, multiplication and division.

The Course

- Introduction to fundamentals of Algebra, including expanded concepts of numbers and order of operations, such as rational number, exponent and radical numbers; and new concepts and rules of variable, algebraic expression, ratio, rate, percent, proportion, quadratic and polynomial, fundamental of linear and quadratic function, and more;
- New math skills of simplifying expressions, factoring, solving linear and quadratic equations and inequalities algebraically or by graphing, graphing lines, inequalities, quadratics and systems, and solving word problems with Algebra.

Teacher's Focus

- Introduce, explain, and apply core algebraic concepts with systemic and comprehensive class notes and examples;
- Emphasize the application of core concepts, and problem solving strategies and skills, through challenging problems with step by step solution;
- Foster students' interest and confidence by guided practice and class discussion.



Hope Chinese School

Syllabus (2023 Summer)			Class: Algebra 1 (AoPS)
Week	DoW	Date	Class Schedule
1	Sat	6/24/23	Chapter 1 Follow the Rules (part 1)
	Tue	6/27/23	Chapter 1 Follow the Rules (part 2)
2	Sat	7/1/23	Chapter 2 x Marks the Spot
	Tue	7/4/23	Chapter 3 One-Variable Linear Equations
3	Sat	7/8/23	Chapter 4 More Variables
	Tue	7/11/23	Chapter 5 Multi-Variable Linear Equations (part 1)
4	Sat	7/15/23	Chapter 5 Multi-Variable Linear Equations (part 2)
	Tue	7/18/23	Chapter 6 Ratios and Percents (part 1)
5	Sat	7/22/23	Chapter 6 Ratios and Percents (part 2)
	Tue	7/25/23	Chapter 7 Proportion (part 1)
6	Sat	7/29/23	Chapter 7 Proportion (part 2)
	Tue	8/1/23	Chapter 8 Graphing Lines (part 1)
7	Sat	8/5/23	Chapter 8 Graphing Lines (part 2)
	Tue	8/8/23	Chapter 9 Introduction to Inequalities
8	Sat	8/12/23	Chapter 10 Quadratic Equations - Part 1 (1)
	Tue	8/15/23	Chapter 10 Quadratic Equations - Part 1 (2)
9			Chapter 11 Special Factorizations (part 1)
			Chapter 11 Special Factorizations (part 2)
10			Chapter 12 Complex Numbers
			Chapter 13 Quadratic Equations - Part 2
11			Chapter 14 Graphing Quadratics (part 1)
			Chapter 14 Graphing Quadratics (part 2)
12			Chapter 15 More Inequalities
			Chapter 16 Functions (part 1)
13			Chapter 16 Functions (part 2)
			Chapter 17 Graphing Functions
14			Chapter 18 Polynomials (part 1)
			Chapter 18 Polynomials (part 2)
15			Chapter 19 Exponents and Logarithms (part 1)
			Chapter 19 Exponents and Logarithms (part 2)
16			Chapter 20 Special Functions (part 1)
			Chapter 20 Special Functions (part 2)

Notes:

- 1 The summer class will be 1.5 hours per class, twice a week, total 16 classes (24 hours)
- 2 No break in the week of Independence Day.
The class on 7/4 can be rescheduled to 7/5 8:00-9:20 PM.
- 3 Students have the option to continue this course in the Fall semester of 2023, if the requirements of minimum number of students is satisfied for the Fall semester.

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Joe Zhou



Subjects taught

Algebra 1 (AoPS), Geometry (AoPS), Algebra 2 (AoPS)

Experience

I have been teaching Algebra 1 (AoPS), Geometry (AoPS), and Algebra 2 (AoPS) courses at Hope Chinese School for over three years.

I have also been working at C2 Education as math tutor for two and a half years, including subjects of Pre-algebra, Algebra 1 and 2, Geometry, Pre-calculus, SAT and ACT Math.

Education

MBA (Kellogg School of Management)

MS Computer Science (University of Illinois at Chicago)

BA Biomedical Engineering (Tsinghua University)

My passion is helping students achieve their full potential.

Are You Ready?

Introduction to Algebra, by R. Rusczyk

This diagnostic test consists of two parts, **Fundamentals** and **Problem Solving**. If your student can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, then the student is ready for the Art of Problem Solving textbook **Introduction to Algebra**.

If the student cannot solve more than 80% of the **Fundamentals** section, then the student should consider our **Prealgebra** text. If the student solves nearly all of the **Fundamentals** but has a great deal of difficulty with the **Problem Solving** problems, then the student should consider reviewing our **Prealgebra** text, which will help the student develop the maturity and problem-solving skills needed for our **Introduction to Algebra** text.

We recommend using the following process in administering this diagnostic:

Step 1: The student should attempt all of the questions below without a calculator and without any help. There is no time limit.

Step 2: Check the student's responses using the answer key at the end of this document.

Step 3: The student should be given a second chance on the problems that they answered incorrectly.

Fundamentals

1. Working with Variables.

- (a) Expand the product $9(3x + 7)$.
- (b) Simplify $7a - 5b + 3(6a + b)$.
- (c) Simplify $a - (-7a - 3)$.
- (d) Simplify $3(5 - 2r) - 2(-3r + 1)$.

2. Fractions, Decimals, and Basic Percents.

- (a) Convert 0.6144 to a fraction in simplest form.
- (b) Convert $17/40$ to a decimal.
- (c) 17 is what percent of 20?
- (d) What is $\frac{9}{5} - \frac{4}{3}$?
- (e) What is $\frac{3/6}{2/5}$ in simplest form?

3. **Linear Equations.** Solve each of the following equations:

(a) $3r - 4 = 16 - 7r$

(b) $\frac{2x - 3}{5} = \frac{4 - 3x}{7}$

(c) $2 - \frac{t}{4} = 3\left(5 - \frac{t}{6}\right)$

4. **Exponent Laws.** Express each of the following as a power of 2:

(a) $2^7 \cdot 2^6$

(b) $\frac{2^{13}}{2^5}$

(c) $(2^4)^3$

(d) $2^3 \cdot \left(\frac{2^7}{2^2}\right)^3$

5. **Ratio and Rates.**

- (a) The ratio of boys to girls at a summer camp is 4 to 5. If the total number of students at the camp is 108, then how many boys are at the camp?
- (b) The ratio of teachers to students in a particular school is 1 to 11. The ratio of female students to the total number of students is 4 to 9. If there are 396 female students, then how many teachers are there?
- (c) A train is traveling 1 mile every 75 seconds. If the train continues at this rate, then how far will it travel in two hours?

6. **Square Roots.** Simplify each of the following as much as possible:

(a) $\sqrt{81}$

(b) $\sqrt{144}$

(c) $\sqrt{1\frac{7}{9}}$

(d) $\frac{\sqrt{540}}{\sqrt{3}}$

Problem Solving

7. What is the value of the sum $5 + 10 + 15 + \cdots + 95 + 100$?
8. Two-fifths of the students in Central Middle School are boys. One-third of the girls have blond hair and one-quarter of the boys have blond hair. What fraction of the students in Central Middle School have blond hair?
9. Kayla adds the same number to both the numerator and denominator of the fraction $\frac{1}{10}$. Her resulting fraction equals $\frac{2}{3}$. What number did she add to both the numerator and denominator of her original fraction?
10. Five workers together can build a road in 20 days. Suppose every worker works at the same rate. Three workers work on the road for 10 days before eleven more workers join them. How much longer will it take the fourteen workers to finish the road?
11. In rectangle $ABCD$, point X is the midpoint of \overline{AD} and Y is the midpoint of \overline{CD} . What fraction of the area of the rectangle is enclosed by $\triangle AXY$?
12. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

Don't look at the next page until you've attempted all the problems!