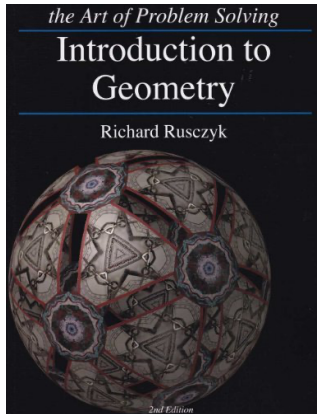


Geometry (AoPS) – Class Description



Textbook and Workbook

Introduction to Geometry, 2nd Edition, by Richard Rusczyk, the Art of Problem Solving series.

Prerequisite

For eight graders or above, or advanced seven graders who have mastered basic Algebra. Students should have passed the course of Algebra a, and be familiar with fundamental concepts of Algebra, such as binomials, quadratics, polynomials, radicals, exponents, rational, linear equation, system of equations, inequalities, numbered line, Cartesian plane, etc., and be familiar with basic skills for solving problems with Algebra, such as distribution, factoring, substitution, simplification, graphing lines and quadratics, etc.

The Course

- Systematic introduction to geometry, including basic to advanced topics such as points, angles, lines, planes, area, perimeter, similar triangles, congruent triangles, right triangles, quadrilaterals, polygons, circles, power of a point, three-dimensional geometry, transformations, introductory trigonometry, laws of logic, and more;
- The course is focused on introduction of fundamental vocabularies, concepts and axioms of geometry, as well as problem solving strategies and techniques, to inspire the students to explore and develop new ideas. Early on in getting the feel and mindset for the proof based approach of geometry paves the way as the students work their way through the journey of math learning.

Teacher's Focus

- To introduce fundamental vocabularies, concepts and axioms of geometry, and problem solving strategies and techniques;
- Emphasize the application of core concepts, and problem solving strategies and skills, through challenging problems with step by step solution;
- Foster students' interest and confidence by class discussion and problem solving related to real life.

Contents

How to Use This Book	iii
----------------------	-----

Acknowledgements	vii
------------------	-----

1	What's in a Name?	1
1.1	Why Names and Symbols?	2
1.2	Points, Lines, and Planes	2
1.3	Round and Round	5
1.4	Construction: Copy a Segment	7
1.5	The Burden of Proof	10
1.6	Summary	10

2	Angles	13
2.1	What is an Angle?	13
2.2	Measuring Angles	14
2.3	Straight and Vertical Angles	21
2.4	Parallel Lines	24
2.5	Angles in a Triangle	31
2.6	Exterior Angles	36
2.7	Parallel Lines Revisited	39
2.8	Summary	42

3	Congruent Triangles	49
3.1	Introduction	49
3.2	SSS Congruence	50
3.3	SAS Congruence	55

CONTENTS

3.4	ASA and AAS Congruence	59
3.5	SSA Not-Necessarily Congruence	63
3.6	Isosceles and Equilateral Triangles	65
3.7	Construction: Equilateral Triangle and Perpendicular Bisector	72
3.8	Summary	73

4 Perimeter and Area **81**

4.1	Perimeter	81
4.2	Area	83
4.3	Same Base/Same Altitude	88
4.4	Summary	92

5 Similar Triangles **99**

5.1	What is Similarity?	99
5.2	AA Similarity	101
5.3	SAS Similarity	108
5.4	SSS Similarity	113
5.5	Using Similarity in Problems	114
5.6	Construction: Angles and Parallels	121
5.7	Summary	124

6 Right Triangles **133**

6.1	Pythagorean Theorem	134
6.2	Two Special Right Triangles	140
6.3	Pythagorean Triples	147
6.4	Congruence and Similarity Revisited	152
6.5★	Heron's Formula	156
6.6	Construction: Perpendicular Lines	161
6.7	Summary	162

7 Special Parts of a Triangle **170**

7.1	Bisectors	170
7.2	Perpendicular Bisectors of a Triangle	173
7.3	Angle Bisectors of a Triangle	177

7.4	Medians	183
7.5	Altitudes	188
7.6★	Challenging Problems	192
7.7	Construction: Bisectors	197
7.8	Summary	199

8 **Quadrilaterals** **206**

8.1	Quadrilateral Basics	206
8.2	Trapezoids	208
8.3	Parallelograms	215
8.4	Rhombi	219
8.5	Rectangles	221
8.6	Squares	224
8.7	If and Only If	227
8.8★	Quadrilateral Problems	232
8.9	Summary	236

9 **Polygons** **244**

9.1	Introduction to Polygons	244
9.2	Angles in a Polygon	246
9.3	Polygon Area	249
9.4	Polygon Problems	254
9.5	Construction: Regular Polygons	257
9.6	Summary	260

10 **Geometric Inequalities** **266**

10.1	Sides and Angles of a Triangle	266
10.2	Pythagoras – Not Just For Right Triangles?	270
10.3	The Triangle Inequality	274
10.4	Summary	280

11 **Circles** **284**

11.1	Arc Measure, Arc Length, and Circumference	284
11.2	Area	290

CONTENTS

11.3	Funky Areas	294
11.4	Summary	299

12	Circles and Angles	304
12.1	Inscribed Angles	304
12.2	Angles Inside and Outside Circles	310
12.3	Tangents	316
12.4	Problems	322
12.5	Construction: Tangents	327
12.6	Summary	330

13	Power of a Point	340
13.1	What is Power of a Point?	340
13.2	Power of a Point Problems	345
13.3	Summary	351

14	Three-Dimensional Geometry	356
14.1	Planes	356
14.2	Prisms	360
14.3	Pyramids	366
14.4	Regular Polyhedra	370
14.5	Summary	374

15	Curved Surfaces	380
15.1	Cylinders	380
15.2	Cones	385
15.3	Spheres	390
15.4	Problems	394
15.5	Summary	402

16	The More Things Change...	408
16.1	Translations	408
16.2	Rotations	411

16.3	Reflections	413
16.4	Dilation	418
16.5	Changing the Question	422
16.6	Construction: Transformations	427
16.7	Summary	429

17	Analytic Geometry	435
17.1	Lines	437
17.2	Circles	444
17.3	Basic Analytic Geometry Problems	446
17.4	Proofs with Analytic Geometry	452
17.5	Distance Between a Point and a Line	461
17.6	Advanced Analytic Geometry Problems	466
17.7	Summary	473

18	Introduction to Trigonometry	478
18.1	Trigonometry and Right Triangles	478
18.2	Not Just For Right Triangles	488
18.3	Law of Sines and Law of Cosines	494
18.4	Summary	500

19	Problem Solving Strategies in Geometry	505
19.1	The Extra Line	505
19.2	Assigning Variables	510
19.3	Proofs	514
19.4	Summary	521

Hints to Selected Problems	525
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Index	552
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Subjects taught

Algebra 1 (AoPS), Geometry (AoPS), Algebra 2 (AoPS)

Experience

I have been teaching Algebra 1 (AoPS), Geometry (AoPS), and Algebra 2 (AoPS) courses at Hope Chinese School for over three years.

I have also been working at C2 Education as math tutor for two and a half years, including subjects of Pre-algebra, Algebra 1 and 2, Geometry, Pre-calculus, SAT and ACT Math.

Education

MBA (Kellogg School of Management)

MS Computer Science (University of Illinois at Chicago)

BA Biomedical Engineering (Tsinghua University)

My passion is helping students achieve their full potential.

Are You Ready?

Introduction to Geometry, by R. Rusczyk

If you've mastered arithmetic, fractions, and the basic algebraic concepts illustrated in the problems below, you are ready for the Art of Problem Solving text, **Introduction to Geometry**. (Answers to these problems can be found on the page following the test questions.)

1. Solving linear equations. Sample questions:

(a) Find x : $31x + 24 = 365$.

(b) Find n : $7n - 4 = 2n + 16$.

2. Simplifying fractions containing algebraic expressions. Reduce the following fractions:

(a) $\frac{3x + 6}{3}$.

(b) $\frac{n(n - 1)}{n(n + 1)(r - 1)}$.

3. Addition and subtraction of quotients with different algebraic denominators. Write each of the following as a single fraction in simplest terms:

(a) $\frac{1}{mn} + \frac{1}{m(2n - 2)}$.

(b) $\frac{r}{r - 1} - \frac{r - 1}{r}$.

4. Multiplication of polynomials and binomials. Expand each of the following:

(a) $(x + 2)(x + 3)$.

(b) $(x + y)(x^2 + 2xy + y^2)$.

(c) $(x - 1)^4$. (Hint: $(x - 1)^4 = (x - 1)(x - 1)^3$.)

5. Solving polynomial equations. Sample questions:

(a) Find x : $x^2 - 18x + 80 = 0$.

(b) Find x : $2x^2 + 5x + 2 = 0$.

(c) Find x : $x^4 - 13x^2 + 36 = 0$. (Hint: let $y = x^2$.)

6. Solving inequalities. Sample questions:

(a) Find the solution set: $2x + 3 \leq 5x - 6$.

(b) Find the solution set: $|x - 3| > 4$.

(c) Find the solution set: $|x - 3| \leq 4$.

Don't look at the next page until you've attempted all the problems!